If $\vec{A}$ and $\vec{B}$ are two differentiable vector functions then

① $\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})$

② $\vec{\nabla}\times(\vec{A}\times\vec{B}) = (\vec{B}\cdot\vec{\nabla})\vec{A} - \vec{B}(\vec{\nabla}\cdot\vec{A})$
$\qquad - (\vec{A}\cdot\vec{\nabla})\vec{B} + \vec{A}(\vec{\nabla}\cdot\vec{B})$

③ $\vec{\nabla}(\vec{A}\cdot\vec{B}) = (\vec{B}\cdot\vec{\nabla})\vec{A} + (\vec{A}\cdot\vec{\nabla})\vec{B} +$
$\qquad \vec{B}\times(\vec{\nabla}\times\vec{A}) + \vec{A}\times(\vec{\nabla}\times\vec{B})$

① Let $\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$

$\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

$= \hat{i}(A_2B_3 - A_3B_2) + \hat{j}(A_3B_1 - A_1B_3)$
$\qquad + (A_1B_2 - A_2B_1)\hat{k}$

$\vec{\nabla}\cdot(\vec{A}\times\vec{B})$

$= \frac{\partial}{\partial x}(A_2B_3 - A_3B_2) + \frac{\partial}{\partial y}(A_3B_1 - A_1B_3)$
$\qquad + \frac{\partial}{\partial z}(A_1B_2 - A_2B_1)$

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$$= B_1\left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right) + B_2\left(\frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x}\right) + B_3\left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right)$$

$$- \left\{ A_1\left(\frac{\partial B_3}{\partial \cancel{z}\, y} - \frac{\partial B_2}{\partial z}\right) + A_2\left(\frac{\partial B_1}{\partial z} - \frac{\partial B_3}{\partial x}\right) + A_3\left(\frac{\partial B_1}{\partial y} - \frac{\partial B_2}{\partial x}\right) \right\}$$

$$= \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - A\cdot(\vec{\nabla}\times\vec{B})$$

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$\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$, $\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$

$\vec{A}\cdot\vec{B} = A_1B_1 + A_2B_2 + A_3B_3$

$\vec{\nabla}(\vec{A}\cdot\vec{B})$

$= \frac{\partial}{\partial x}(A_1B_1 + A_2B_2 + A_3B_3)\hat{i}$

$+ \frac{\partial}{\partial y}(A_1B_1 + A_2B_3 + A_3B_3)\hat{j} + \frac{\partial}{\partial z}(A_1B_1 + A_2B_2 + A_3B_3)\hat{k}$

$(\vec{B}\cdot\vec{\nabla})\vec{A}$

$= \left(B_1\frac{\partial}{\partial x} + B_2\frac{\partial}{\partial y} + B_3\frac{\partial}{\partial z}\right)\vec{A}$

$= \left(B_1\frac{\partial A_1}{\partial x} + B_2\frac{\partial A_1}{\partial y} + B_3\frac{\partial A_1}{\partial z}\right)\hat{i}$

$+ \left(B_1\frac{\partial A_2}{\partial x} + B_2\frac{\partial A_2}{\partial y} + B_3\frac{\partial A_2}{\partial y}\right)\hat{j} + \left(B_1\frac{\partial A_3}{\partial x} + B_2\frac{\partial A_3}{\partial y} + B_3\frac{\partial A_3}{\partial z}\right)\hat{k}$

$\cancel{(\vec{A}\cdot\vec{B})}$ $(\vec{A}\cdot\vec{\nabla})\vec{B}$

$= \left(A_1\frac{\partial B_1}{\partial x} + A_2\frac{\partial B_1}{\partial y} + A_3\frac{\partial B_1}{\partial z}\right)\hat{i}$

$+ \left(A_1\frac{\partial B_2}{\partial x} + A_2\frac{\partial B_2}{\partial y} + A_3\frac{\partial B_3}{\partial y}\right)\hat{j} + \left(A_1\frac{\partial B_3}{\partial x} + A_2\frac{\partial B_3}{\partial y} + A_3\frac{\partial B_3}{\partial z}\right)\hat{k}$

$(\vec{B}\cdot\vec{\nabla})\vec{A} + (\vec{A}\cdot\vec{\nabla})\vec{B}$

$= \frac{\partial}{\partial x}(A_1B_1)\hat{i} + \frac{\partial}{\partial y}(A_2B_2)\hat{j} + \frac{\partial}{\partial z}(A_3B_3)\hat{k}$

$+ \left(B_2\frac{\partial A_1}{\partial y} + B_3\frac{\partial A_1}{\partial z} + A_2\frac{\partial B_1}{\partial y} + A_3\frac{\partial B_1}{\partial z}\right)\hat{i}$

$+ \left(B_1\frac{\partial A_2}{\partial x} + B_3\frac{\partial A_2}{\partial y} + A_1\frac{\partial B_2}{\partial x} + A_3\frac{\partial B_2}{\partial y}\right)\hat{j}$

$+ \left(B_1\frac{\partial A_3}{\partial x} + B_2\frac{\partial A_3}{\partial y} + A_1\frac{\partial B_3}{\partial x} + A_2\frac{\partial B_3}{\partial y}\right)\hat{k}$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right) + \hat{j}\left(\frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x}\right) + \hat{k}\left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right)$$

$$\vec{B} \times (\vec{\nabla} \times \vec{A}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ B_1 & B_2 & B_3 \\ \frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z} & \frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x} & \frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y} \end{vmatrix}$$

$$= \hat{i}\left(B_2 \frac{\partial A_2}{\partial x} - B_2 \frac{\partial A_1}{\partial y} - \frac{\partial A_1}{\partial z} B_3 + B_3 \frac{\partial A_3}{\partial x}\right)$$

$$+ \hat{j}\left(B_3 \frac{\partial A_3}{\partial y} - B_3 \frac{\partial A_2}{\partial z} - B_1 \frac{\partial A_2}{\partial x} + B_1 \frac{\partial A_1}{\partial y}\right)$$

$$+ \hat{k}\left(B_1 \frac{\partial A_1}{\partial z} - B_1 \frac{\partial A_3}{\partial x} - B_2 \frac{\partial A_3}{\partial y} + B_2 \frac{\partial A_2}{\partial z}\right)$$

$$\vec{A} \times (\vec{\nabla} \times \vec{B})$$

$$= \hat{i}\left(A_2 \frac{\partial B_2}{\partial x} - A_2 \frac{\partial B_1}{\partial y} - \frac{\partial B_1}{\partial z} A_3 + A_3 \frac{\partial B_3}{\partial x}\right)$$

$$+ \hat{j}\left(A_3 \frac{\partial B_3}{\partial y} - A_3 \frac{\partial B_2}{\partial z} - A_1 \frac{\partial B_2}{\partial x} + A_1 \frac{\partial B_1}{\partial y}\right)$$

$$+ \hat{k}\left(A_1 \frac{\partial B_1}{\partial z} - A_1 \frac{\partial B_3}{\partial x} - A_2 \frac{\partial B_3}{\partial y} + A_3 \frac{\partial B_2}{\partial z}\right)$$

$$\vec{A} \times (\vec{\nabla} \times \vec{B}) + \vec{B} \times (\vec{\nabla} \times \vec{A})$$

$$= \cancel{\frac{\partial}{\partial x}(A_2 B_2)\hat{i}} = \frac{\partial}{\partial x}(A_2 B_2 + A_3 B_3)\hat{i}$$

$$+ \frac{\partial}{\partial y}(A_1 B_1 + A_3 B_3)\hat{j} + \frac{\partial}{\partial z}(A_1 B_1 + A_2 B_2)\hat{k}$$

$$- \Big\{\left(B_2 \frac{\partial A_1}{\partial y} + \frac{\partial A_1}{\partial z} B_3 + A_2 \frac{\partial B_1}{\partial y} + \frac{\partial B_1}{\partial z} A_3\right)\hat{i} +$$

$$\left(B_3 \frac{\partial A_2}{\partial z} + B_1 \frac{\partial A_2}{\partial x} + A_3 \frac{\partial B_2}{\partial z} + A_1 \frac{\partial B_2}{\partial x}\right)\hat{j} + \Big(B_1 \frac{\partial A_3}{\partial x} + B_2 \frac{\partial A_3}{\partial y}$$

$$+ A_1 \frac{\partial B_3}{\partial x} + A_2 \frac{\partial B_3}{\partial y}\Big)\hat{k} \Big\}$$

$$(\vec{B}\cdot\vec{\nabla})\vec{A} + (\vec{A}\cdot\vec{\nabla})\vec{B} + \vec{B}\times(\vec{\nabla}\times\vec{A}) + \vec{A}\times(\vec{\nabla}\times\vec{B})$$

$$= \frac{\partial}{\partial x}(A_1B_1 + A_2B_2 + A_3B_3)\hat{i} + \frac{\partial}{\partial y}(A_1B_1 + A_2B_2 + A_3B_3)\hat{j} + \frac{\partial}{\partial z}(A_1B_1 + A_2B_2 + A_3B_3)\hat{k}$$

$$= \vec{\nabla}(\vec{A}\cdot\vec{B})$$